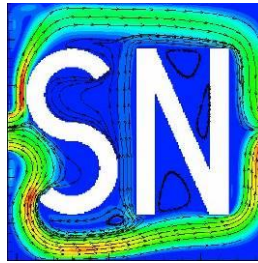


Mach 4.5 Turbulent Flow Over Flat Plate
SmartNumerics Simulation Solutions Inc.



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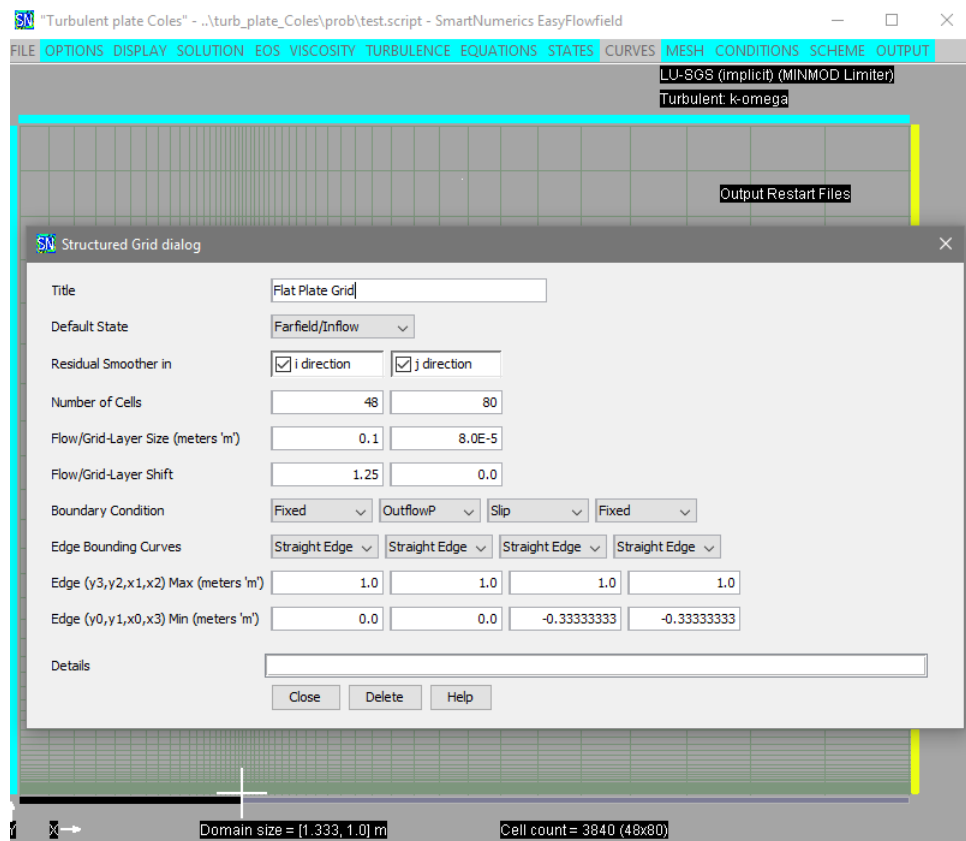


Fig. 1: Grid and grid parameters used for Mach 4.5 turbulent flow over flat plate.

This validation test case is based on the experimental work of Coles [1, 2]. Figure 1 displays the grid and the structured-grid dialog parameters used to simulate turbulent flow of air over a flat plate using the ideal-gas equation of state. Freestream conditions are imposed at the left and top boundaries and the freestream static pressure is imposed at the right boundary. The no-slip wall (grey) is imposed using a second dialog.

Table 1: Test conditions used by Coles [1]

| Test # | 25 | 22 | 61 | 76 | 62 |
|----------|--------------------|--------------------|--------------------|--------------------|--------------------|
| Trip | Fence | Fence | Sand | Jets | Jets |
| Mach | 4.512 | 4.554 | 4.545 | 4.504 | 4.544 |
| Po mm Hg | 1480 | 2920 | 3010 | 3010 | 3020 |
| To °R | 552 | 554 | 561 | 564 | 563 |
| Po Pa | 197316.56 | 389300.24 | 401299.22 | 401299.22 | 402632.44 |
| To °K | 306.6666667 | 307.7777778 | 311.6666667 | 313.3333333 | 312.7777778 |
| P Pa | 671.6568179 | 1257.806407 | 1311.116522 | 1379.691223 | 1317.104712 |
| T °K | 60.46709623 | 59.78841102 | 60.73710157 | 61.95782944 | 60.97523360 |

Table 1 contains information extracted from Coles [1]. The total conditions in °R and mm Hg provided by Coles [1] have been converted to freestream static conditions expressed in °K and Pascals. Note that the total pressure for test 25 is approximately half of the total pressure specified for the other tests. The Mach number and temperature for all the tests are fairly close.

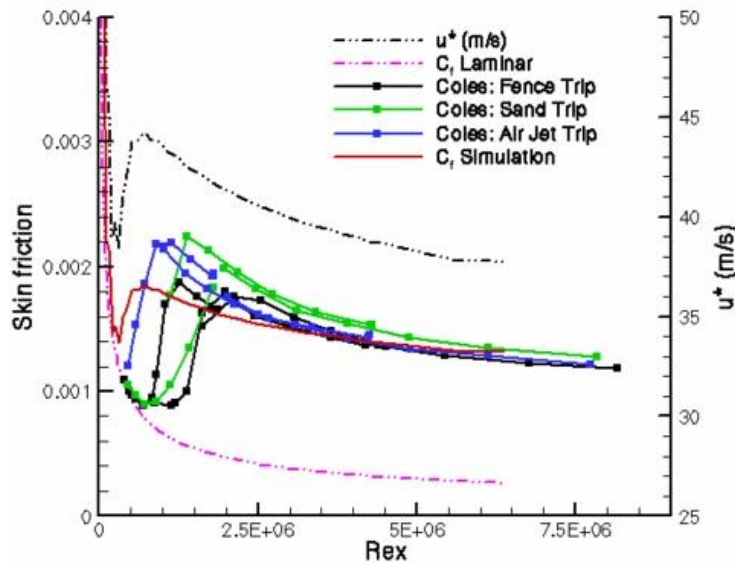
**Fig. 2: Friction vs Reynolds number for Mach 4.5 turbulent flow over plate (k- ω).**

Figure 2 displays a comparison of the friction curve from the simulation to various experimental friction curves. The friction velocity (u^*) is also plotted. Here

$$Re_x = \frac{\rho_\infty u_\infty x}{\mu_\infty}$$

is the Reynolds number a specified distance x downstream of the leading edge of the plate, where μ_∞ is the far-field molecular viscosity, u_∞ is the far-field velocity, and ρ_∞ is the far-field density. The friction from the simulation was computed using

$$c_f = 2 \frac{\rho_w (u^*)^2}{\rho_\infty U_\infty^2}$$

where ρ_w is the density at the wall.

The test conditions were taken from experiment number 25 of Coles [1]. That is, air with a far field static pressure of 671.7 Pa and static temperature of 60.47 °K moving at Mach 4.512 travels over a smooth flat plate. Molecular viscosity was computed from temperature using the Sutherland formula. The $k-\omega$ turbulence model of Wilcox [3] was used with a compressibility adjustment but without a wall function. Turbulence was damped upstream of the plate leading edge to improve stability. The Roe flux was used in this simulation, but the HLLC flux gave similar results. The MINMOD limiter was used to obtain second order accuracy in space. The (implicit) LU-SGS solver was used with a CFL of 40. The friction curves all follow the Blasius laminar-flow friction correlation

$$c_{f \text{ laminar}} = \frac{0.664}{\sqrt{Re_x}}$$

for a certain distance before moving upward as the flow becomes turbulent. Coles [1,2] used various devices to trip the flow from laminar to turbulent at a point earlier than it would do so naturally. The experimental curves depart at various points depending on whether turbulent flow is tripped by an air jet, a strip of glued sand, or a leading-edge fence consisting of a row of short wire posts normal to the airflow and to the plate. The air jet is most efficient a promoting transition. The friction curve from the simulation departs at the lowest Reynolds number because the $k-\omega$ model was designed with the assumption of fully turbulent flow and usually displays little or no laminar-flow behaviour.

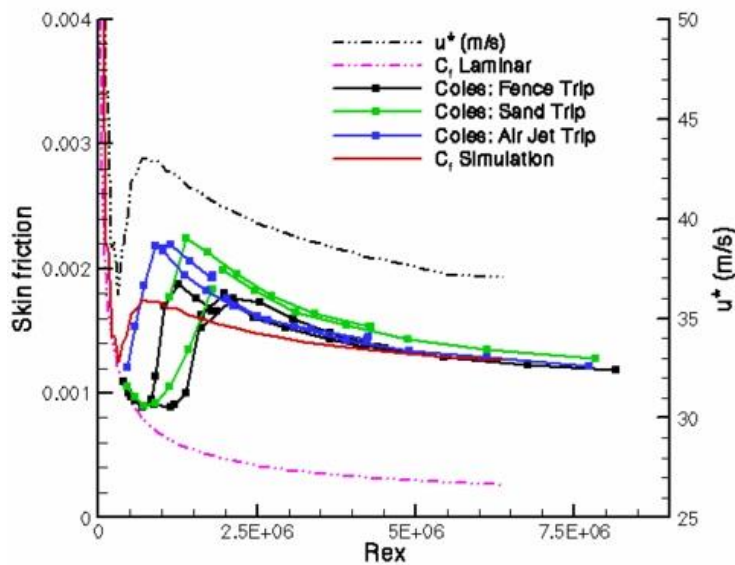


Fig. 3: Friction vs Reynolds number for Mach 4.5 flow over plate (k- ω 2006).

Figure 3 displays the friction curve obtained using the $k-\omega$ 2006 model of Wilcox [4]. In this case, the friction curve from the simulation departs from the laminar friction curve further downstream than seen using the $k-\omega$ turbulence model.

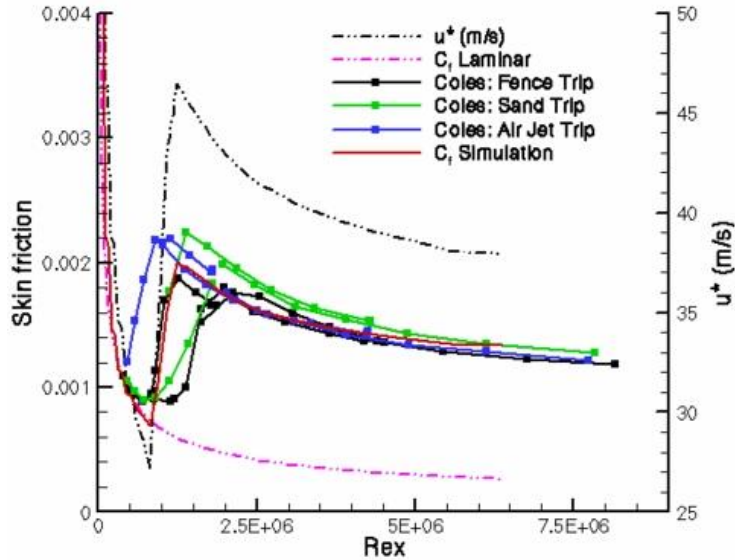


Fig. 4: Friction vs Reynolds number for Mach 4.5 flow with damping (k- ω).

Figure 4 displays an altered friction curve from a k- ω simulation with damping of turbulence imposed upstream of 0.15 meters. The result of the delayed transition to turbulent flow is a good fit to one of the tests where a fence was used to trip turbulence.

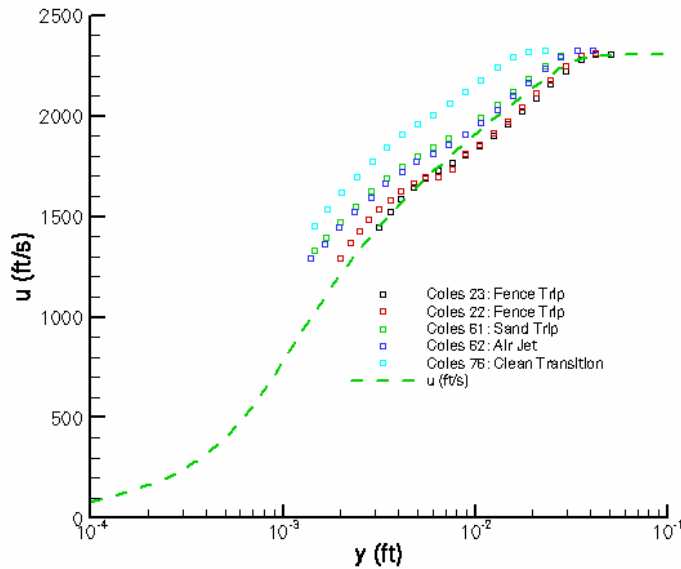


Fig. 5: Velocity vs distance from plate, 1.79 feet down-stream of leading edge (k- ω).

Figure 5 displays a comparison to experiment of velocity versus distance from the plate 1.79 feet down-stream of the plate leading edge. The match is acceptable for tests using a device to trip transition to turbulent flow and poor for a test without forced transition. The k- ω turbulence model was used with damping imposed upstream of the leading edge. As seen in Figures 6 and 7, similar velocity curves were obtained using the k- ω 2006 turbulence model and the SST turbulence model of Menter [5].

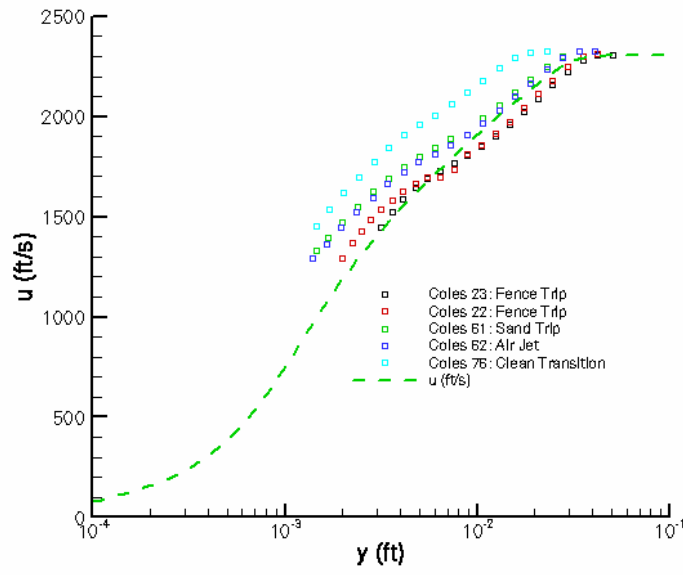


Fig. 6: Velocity vs y , 1.79 feet down-stream of leading edge ($k-\omega$ 2006).

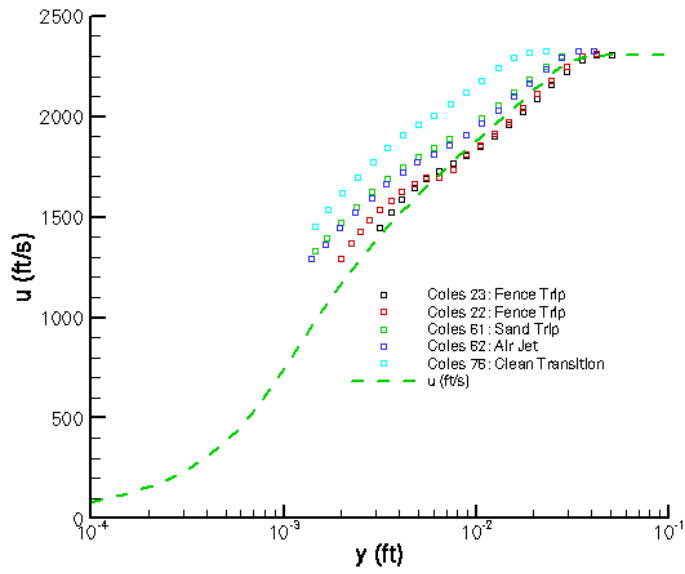


Fig. 7: Velocity vs distance from plate 1.79 feet down-stream of leading edge (SST).

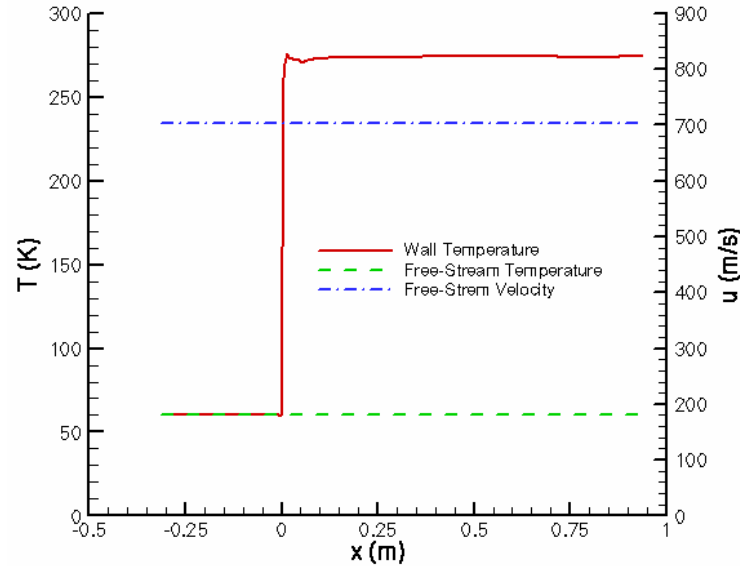


Fig. 8: Temperature and velocity curves for turbulent flow over flat plate (k- ω).

Finally, Figure 8 compares the freestream temperature to the temperature at the wall. For laminar or turbulent flow, if a gas is decelerated to zero velocity by contact with an insulated wall, the gradient in velocity normal to the wall allows small mass transport effects to dissipate some of the heat energy so that it does not reach the wall. These non-isentropic processes reduce the temperature at the wall so that it does not reach the total temperature. According to White [7], this recovery temperature can be approximately computed using

$$T_r = T_\infty \left(1 + r \frac{\gamma - 1}{2} M^2 \right) = T_\infty + r \frac{U_\infty^2}{2C_p}$$

where T_∞ is the freestream static temperature, C_p is the specific heat, γ is the specific heat ratio, M is the Mach number, and r is the recovery factor which is less than one. For turbulent flow, the recovery factor is often approximated as the cube root of the laminar Prandtl number. The laminar Prandtl number was set to 0.713 for this simulation. Assuming a specific heat of 1003.5 J/Kg/°K or using the expression based on Mach number, the recover temperature computed using the above equation is about 281°K. This is slightly above the value of about 274 °K seen in Figure 8. Thus, the correct recovery factor is slightly smaller than the cube root of 0.713.

References

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