Turbulent Flow Over Rough-Walled Flat Plate

SmartNumerics Simulation Solutions Inc.

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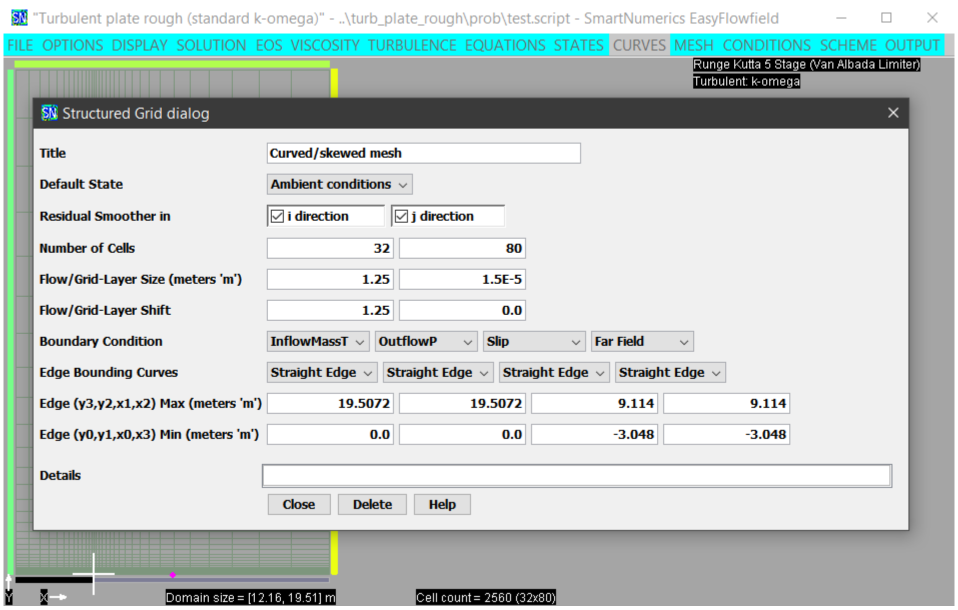

Fig. 1: Grid and wall condition for turbulent transitionally rough flow over plate.

Figure 1 displays the grid used to simulate transitionally rough turbulent flow of air over a rough plate. The wall roughness size was set to 3.028×10^{-4} m. This is also called roughness height, roughness diameter, absolute roughness, and equivalent sand grain diameter (d_{eq}) in the literature. This is a value quantifying the frictional resistance to flow and is not directly relatable to the height of irregularities on a wall. The resistance to flow will depend on the shape, size, and separation of the irregularities (asperities or roughness elements) on the wall. The roughness size is measure of flow resistance and is based on tabulated flow rates through specially treated pipes; closely packed grains of the specified size are glued to the pipe wall and the grains are subsequently coated with more glue.

The far-field pressure, temperature, and velocity were 74,622 Pascals, 273.15° K, and 45.3 m/s, respectively. The molecular viscosity was computed using Sutherlands formula. The simulation was solved using an explicit five-stage Runge-Kutta scheme with enriched w-cycle multigrid. The HLLC flux was used with the van Albada limiter. The k-ω turbulence model of Wilcox [1] was used without a wall function. The ideal-gas equation of state was employed.

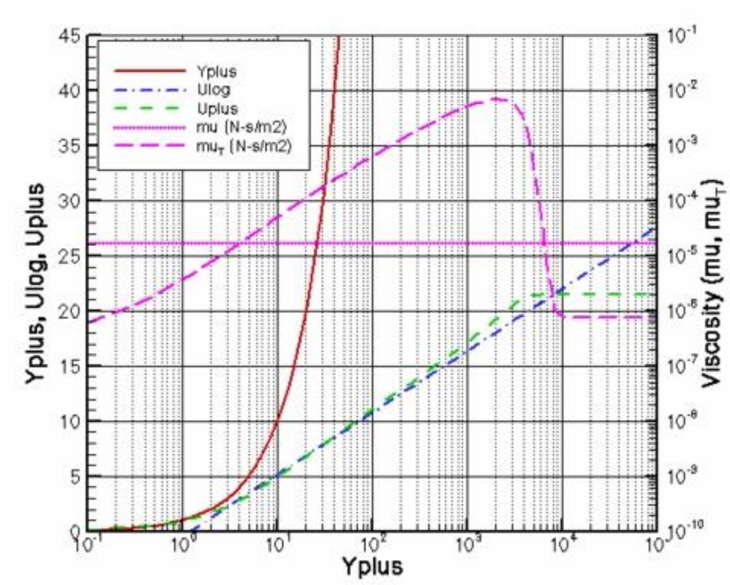

Fig. 2: Plot of u^+ versus y^+ for transitionally rough turbulent flow over flat plate.

Figure 2 displays a plot of $u^+ = u/u^*$ versus y^+ for transitionally rough turbulent flow obtained using the k-ω turbulence model. As expected, this follows the y^+ curve near the wall. Further from the wall it matches the incompressible log law for flow over a rough wall

$$\mathrm{U_{log}} = \frac{1}{\kappa}\ln\left(y^+\right) + B - \Delta B$$

where

$$y^+ = \frac{\rho_w u^* y}{\mu_w}$$

is the normalized distance from the wall and

$$\Delta B = \frac{1}{0.41}\ln\left(1 + 0.238 d_{eq}^+\right)$$

is an offset in u^+. Here ρ_w is the density next to the wall, y is the distance from the wall, u^* is the friction velocity, μ_w is the molecular viscosity at the wall, and

$$d_{eq}^+ = \frac{\rho_w u^* d_{eq}}{\mu_w}$$

is the normalized equivalent sand grain diameter. The flow is in the hydrodynamically-smooth regime if d_{eq}^+ does not exceed 5, is in the transitionally rough regime if d_{eq}^+ exceeds 5, and is less than 70, and is in the-fully rough regime if d_{eq}^+ is 70 or larger. In this case, d_{eq}^+ varies from 47 near the plate leading edge to 29 near the outflow boundary.

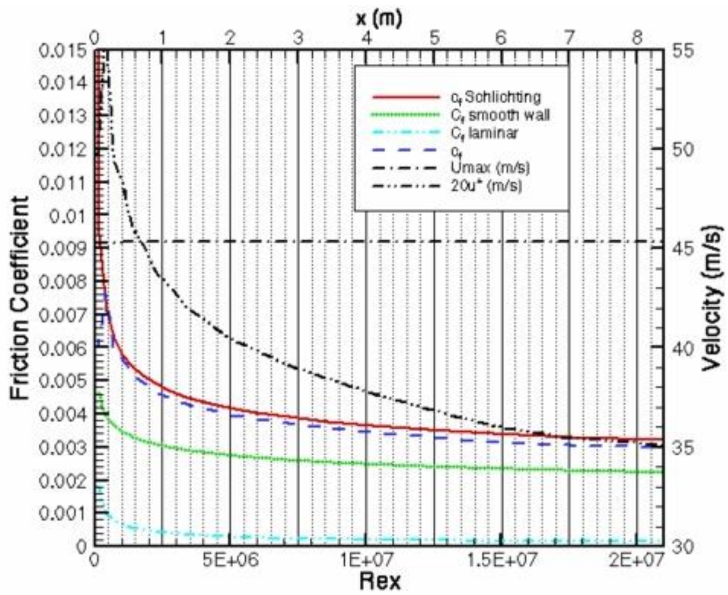

Fig. 3: Friction versus Reynolds number for transitionally rough flow over plate.

Figure 3 displays a plot of skin friction versus Reynolds number

$$\mathrm{Re}_x = \frac{\rho_\infty u_\infty x}{\mu_\infty}$$

based on downstream distance (x) from the plate leading edge. Here μ_∞ is the far-field molecular viscosity, u_∞ is the far-field velocity, and ρ_∞ is the far-field density. The skin friction coefficients are computed using

$$c_f = 2\frac{\rho_w (u^*)^2}{\rho_\infty u_\infty^2}$$

using the local friction velocity, u^*, output by the solver. Here, ρ_w is the local density at the wall.

This curve is slightly above the Schlichting [2] rough-plate friction correlation

$$c_{f\ \mathrm{Schlichtig}} = \left[2.87 + 1.58 \log 10\left(\frac{x}{d_{eq}}\right)\right]^{-2.5},$$

well above the smooth-wall turbulent-flow friction correlation

$$c_{f\ \mathrm{smooth}} = .025(\mathrm{Re}_x)^{-1/7} \quad ,$$

and further above the Blasius laminar-flow friction correlation

$$c_{f\ \mathrm{laminar}} = \frac{0.664}{\sqrt{\mathrm{Re}_x}} \quad .$$

The friction velocity multiplied by 20 is also plotted in Figure 3 along with the velocity near the far-field boundary.

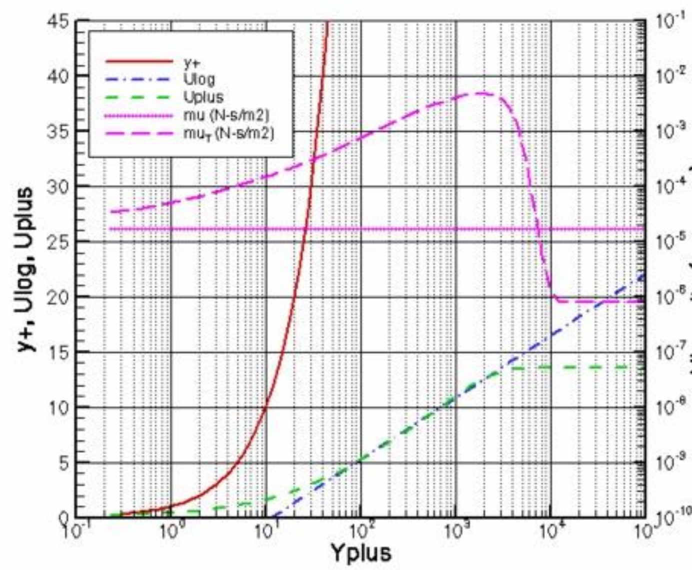

Fig. 4: Plot of u^+ versus y^+ for fully rough turbulent flow over flat plate.

Figure 4 displays a plot of u^+ versus y^+ for fully rough turbulent flow. The simulation employs the same grid but uses an implicit LU-SGS solver. Figure 5 displays friction. The far-field flow velocity is 73 m/s and the roughness size is 1.2677×10^{-3} m. The normalized roughness size varies from 393 at the leading edge to 228 just upstream if the outflow boundary.

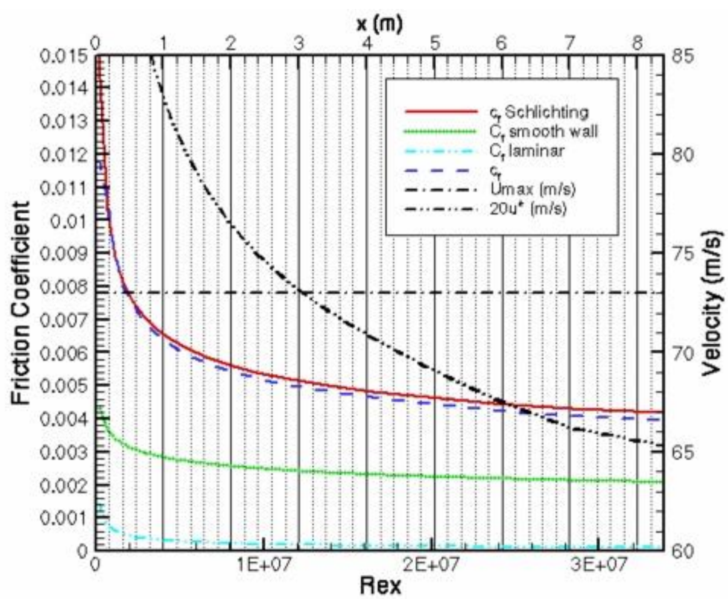

Fig. 5: Friction versus Reynolds number for fully rough turbulent flow over plate.

These validation tests have also been performed using the k-ω 2006 turbulence model of Wilcox [3] with the stress limiter weight set to zero. The friction curves produced by the k-ω 2006 turbulence model are a bit lower than those produced by the k-ω turbulence model.

References

[1] Wilcox, D. C., Turbulence Modeling for CFD, DCW Industries Inc., 1994.
[2] Schlichting, H., Boundary-Layer Theory, 7th Edition, McGraw-Hill, New York, 1979.
[3] Wilcox, D. C., Turbulence Modeling for CFD, 3rd ed., DCW Industries Inc., 2006.